Functions – Dividing Polynomials

Ex. $(x^3 - 6x^2 + 17x - 24) \div (x - 3)$ which could also look like $\frac{x^3-6x^2+17x-24}{x-3}$

The process for dividing polynomials is the same as the way you learned in fourth grade.

Old time Ex. $\frac{215}{4}$ can be found using LONG DIVISION. Remember $\frac{215}{4} = 4\overline{)215}$

$4\overline{)215}$ → $\begin{array}{r}5\\ 4\overline{)215}\end{array}$ (1 Pick a number: *4 times WHAT is less than 21?*) → $\begin{array}{r}5\\ 4\overline{)215}\\ 20\end{array}$ (2 Multiply: *4 X 5 = 20*) → $\begin{array}{r}5\\ 4\overline{)215}\\ -20\\ \hline 1\end{array}$ (3 Subtract: *21 – 20 = 1*) → $\begin{array}{r}5\\ 4\overline{)215}\\ -20\downarrow\\ \hline 15\end{array}$ (4 Bring Down:) → $\begin{array}{r}53\\ 4\overline{)215}\\ -20\\ \hline 15\end{array}$ (Then REPEAT 1 Pick a number:) → $\begin{array}{r}53\\ 4\overline{)215}\\ -20\\ \hline 15\\ 12\end{array}$ (2 Multiply:) → $\begin{array}{r}53\\ 4\overline{)215}\\ -20\\ \hline 15\\ -12\\ \hline 3\end{array}$ (3 Subtract:)

So $\frac{215}{4} = 53\frac{3}{4}$ OR = 53 with a remainder of 3

To divide polynomials, use the same process:

Ex: $\frac{x^3-6x^2+17x-24}{x-3}$ can be written as $x-3\overline{)x^3-6x^2+17x-24}$

$x-3\overline{)x^3-6x^2+17x-24}$

Write the terms of both polynomials from highest degree to lowest degree.

Keep like terms together in the same column.

$\begin{array}{r}x^2\\ x-3\overline{)x^3-6x^2+17x-24}\\ -\ x^3-3x^2\\ \hline -3x^2+17x\end{array}$

1. Pick a number: x times what is $x^3$ ? $x^2$
2. Multiply: $x^2 \cdot (x-3) = x^3 - 3x^2$
3. Subtract: $(x^3 - 6x^2) - (x^3 - 3x^2) = -3x^2$
4. Bring down: the $+17x$

$\begin{array}{r}x^2-3x\\ x-3\overline{)x^3-6x^2+17x-24}\\ -\ x^3-3x^2\\ \hline -3x^2+17x\\ -\ -3x^2+9x\\ \hline 8x-24\end{array}$

1. Pick a number: x times what is $-3x^2$ ? $-3x$
2. Multiply: $-3x \cdot (x-3) = -3x^2 + 9x$
3. Subtract: $(-3x^2 + 17x) - (-3x^2 + 9x) = 8x$
4. Bring down: the $-24$

$\begin{array}{r}x^2-3x+8\\ x-3\overline{)x^3-6x^2+17x-24}\\ -\ x^3-3x^2\\ \hline -3x^2+17x\\ -\ -3x^2+9x\\ \hline 8x-24\\ -\ 8x-24\\ \hline 0\end{array}$

1. Pick a number: x times what is $8x$ ? $8$
2. Mulitiply: $8 \cdot (x-3) = 8x - 24$
3. Subtract: $(8x - 24) - (8x - 24) = 0$

Therefore: $\frac{x^3-6x^2+17x-24}{x-3} = x^2 - 3x + 8$

Ex 2: $\frac{x^3-5x+8}{x-2}$

Note: There is no $x^2$ term. We need one, so we don't get confused later. So we'll artificially add one that doesn't change the polynomial.

$$x-2\overline{)\,x^3+0x^2-5x+8}$$

$$\begin{array}{r} x^2 \\ x-2\overline{)\,x^3+0x^2-5x+8} \\ -\,(x^3-2x^2) \\ \hline 2x^2-5x \end{array}$$

$$\begin{array}{r} x^2+2x \\ x-2\overline{)\,x^3+0x^2-5x+8} \\ -\,(x^3-2x^2) \\ \hline 2x^2-5x \\ -\,(2x^2-4x) \\ \hline -x+8 \end{array}$$

$$\begin{array}{r} x^2+2x-1 \\ x-2\overline{)\,x^3+0x^2-5x+8} \\ -\,(x^3-2x^2) \\ \hline 2x^2-5x \\ -\,(2x^2-4x) \\ \hline -x+8 \\ -\,(-x+2) \\ \hline 6 \end{array}$$

So $\frac{x^3-5x+8}{x-2} = x^2+2x-1 \text{ with a remainder of } 6$ OR

$$\frac{x^3-5x+8}{x-2} = x^2+2x-1+\frac{6}{x-2}$$

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Ex 3: $\frac{x^4-4x^3+6x^2-20x+5}{x^2+5}$

$$\begin{array}{r} x^2 \\ x^2+5\overline{)\,x^4-4x^3+6x^2-20x+5} \\ -\,(x^4+5x^2) \\ \hline -4x^3+x^2-20x \end{array}$$

$$\begin{array}{r} x^2-4x \\ x^2+5\overline{)\,x^4-4x^3+6x^2-20x+5} \\ -\,(x^4+5x^2) \\ \hline -4x^3+x^2-20x \\ -\,(-4x^3-20x) \\ \hline x^2+5 \end{array}$$

$$\begin{array}{r} x^2-4x+1 \\ x^2+5\overline{)\,x^4-4x^3+6x^2-20x+5} \\ -\,(x^4+5x^2) \\ \hline -4x^3+x^2-20x \\ -\,(-4x^3-20x) \\ \hline x^2+5 \\ -\,(x^2+5) \\ \hline 0 \end{array}$$

Note: It's really important to put terms in the proper columns

So $\frac{x^4-4x^3+6x^2-20x+5}{x^2+5} = x^2-4x+1$